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with the compliments of
the author.

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NOTES ON PHYSIOLOGICAL OPTICS.

III and IV.

By W. LeConte Stevens.

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No. III.

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[FROM THE AMERICAN JOURNAL OF SCIENCE, VOL. XXIII, APRIL, 1882.]

ART. XXXI.—*Notes on Physiological Optics*, No. III; by W. LECONTE STEVENS.

1. THEORY OF ASSOCIATED MUSCULAR ACTION.

IN previous articles[1] it has been shown that the current theory of binocular perspective applied to the stereoscope is not only incapable of accounting for many observed facts but unsatisfactory even when the visual lines are convergent; that the apparent position of points in the stereoscopic field of view cannot be determined by any mathematical formula or accurately represented by diagram; and that this impossibility is due to physiological conditions attendant upon the abnormal use of the eyes.

That convergence of visual lines should be deemed a necessary condition in binocular vision, or, if not necessary, at least invariably present, and that it should be assumed, either expressly or implicitly, in most, if not all, of our text-books, is doubtless due to the fact that theoretically the stereoscope is intended to reproduce as nearly as possible the conditions attendant upon normal binocular vision of the objects pictured. Sir David Brewster gave especial emphasis to the theory that every point in the field of view is determined by triangulation with visual lines, elaborating this idea first in an article,[2] published in 1844, "On the Knowledge of Distance given by

Binocular Vision," and subsequently in his book on the Stereoscope,[3] published in 1856. Professor W. B. Rogers contributed to this Journal in 1855 and 1856 a series of most interesting articles on Binocular Vision,[4] in which he determined mathematically what should be the form of the resultant curve when images of dissimilar lines are binocularly combined, each point of each curve being determined by intersection of visual lines. I have performed most of Professor Rogers' experiments successfully with optic divergence. The same remark applies to those of Professor LeConte, wherever the combination is effected by diminution of the convergence that would be natural in ordinary vision.

In 1862, Professor C. F. Himes, at that time of Troy University, published an article[5] in which he criticised Brewster's theory and called attention to the modification necessitated by the possibility of stereoscopic vision with divergence of visual lines. This article, which was sent me by the author immediately after the publication of my paper in November last in this Journal, establishes his priority in this country in discovering the possibility of stereoscopic vision by this method. In 1861 two Germans, Rollet and Becker, published a method of combining similar images by optic divergence, the possibility of doing this having been already mentioned by Burckhardt.[6] Professor Himes' observations were made without any knowledge of what had just been accomplished in Germany. My own discovery was likewise independent; finding however that others had preceded me, and that prisms had been often employed to test the external rectus muscles, I claimed nothing on this ground, but devoted my attention to the analysis of vision by optic divergence; for on this little had been written. My rejection of Brewster's theory and the expression of my conviction that in associated muscular action is to be found the explanation of what has generally been referred to intersection of visual lines, was publicly made in my papers, read June 6th and Aug. 19th, 1881, the latter having since been published in this Journal. It is a source of satisfaction now to find, in the London Lancet, of Oct. 22d, and Dec. 31st, 1881, two able papers written by Brigade Surgeon Tyler Oughton, of the English army, who with no knowledge of what had been expressed by me, reaches conclusions closely akin to my own, substituting for the current theory that of "muscular consent," and rejecting the theory of corresponding retinal points.

It is but right to add that this theory was virtually stated by Professor Huxley[12] in 1868, and in such a way as quite plainly to indicate its applicability to the phenomena of optic divergence. That he should have been satisfied with a brief statement that has passed almost unnoticed, instead of elabo-

rating it in refutation of Brewster's theory, was probably due to the fact that the fallacy and popularity of the latter, in its application to the stereoscope, had not been brought especially to his attention.

2. Relation between Different Elements of Binocular Perspective.

The fact that in all stereoscopic vision there is necessarily an interruption of the usual relation between the axial and focal adjustments of the eyes was first noticed by Professor W. B. Rogers, who makes however no reference to the production of any disturbance of perspective, as noticed by myself. In the articles already published it has been shown that even when the stereograph is so constructed as to exclude to the utmost the ordinary elements of perspective, there are left still three to consider. These are—

I. The optic angle, positive or negative, enclosed by the visual lines and interpreted through the sensation of contraction or relaxation in the rectus muscles of the eyeballs.

II. The focal adjustment, interpreted through the sensation of contraction or relaxation in the ciliary muscle encircling the crystalline lens.

III. The visual angle, subtended by the diameter of the object regarded, and interpreted by recognition of the retinal area impressed but instantly and unconsciously referred to the external object.

With a view to finding, if possible, what relation these three elements bear to each other in abnormal vision like that in the stereoscope, I constructed a modification of the instrument originally devised by Wheatstone. Upon a cubical block,

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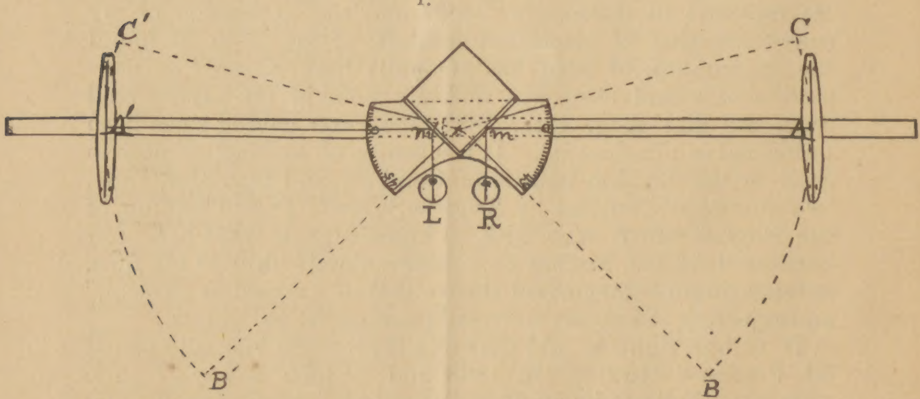

(fig. 1) two plane mirrors, m and m', were cemented, and a pair of arms were attached to carry the conjugate pictures, A and

A′. These arms move upon a pivot, each through an arc of 60°, under a divided circle. When so adjusted that the angle of incidence on each side is 45°, the direction of the reflected rays is such as to necessitate parallelism of visual lines for those which come from the centers of A and A′ respectively. If the arms are pulled forward for example to B and B′, the angle of incidence becomes such that the eyes must be made to roll inward to retain binocular combination of images; if pushed back toward C and C′, divergence of visual lines is necessitated. The value of the optic angle, positive for convergence, negative for divergence, is obtained, with but trifling error, from the circle. On each side let the picture be kept at a fixed distance, while the eyes are as near as possible to the mirrors, for example, so that $Am + mR = 50^{cm}$. For this distance, in normal vision, the value of the optic angle would be 7° 20′. If each arm therefore be pulled forward 3° 40′, the binocular image appears in full relief about 50^{cm} in front. Shifting the head slightly to one side, the rays from one picture are reflected by a single mirror into both eyes, and the image now appears flat at the distance of 50^{cm}. The apparent distance of this flat image is obviously independent of the relation between the arms of the instrument, and serves as a standard of comparison, the optic angle, focal adjustment and visual angle all conducing to the same judgment of distance.

Modifying slightly the formula hitherto employed, we have, for the distance, D, of the optic vertex from each eye, determined by intersection of visual lines,

$$D = \tfrac{1}{2}i \operatorname{cosec} \tfrac{1}{2}\alpha,$$

where i is the interocular distance, and α the optic angle. If this equation be expressed as a curve, fig. 2, taking values of α for abscissas and values of D for ordinates, the axis of ordinates is obviously an asymptote.

Let the arms now be pulled forward until $\alpha = 37° 20'$. The corresponding value of D is 10^{cm}, while the visual angle is unchanged and the focal adjustment, if perfectly distinct vision be secured, must still be for a distance of 50^{cm}. These two elements therefore tend to counteract the suggestion due to strong convergence, and the image appears perhaps 15^{cm} or 20^{cm} distant. Its apparent diameter varies directly as the estimated distance, and is diminished to ·3 or ·4 of the original diameter. The influence of axial convergence, though partially counteracted, preponderates over that of the other elements in determining the judgment.

Let the arms now be pushed back until $\alpha = -5°$. The theoretic value of D is negative and hence physically impossible, but practically the contraction of the external rectus muscles

produces the impression of continued recession in a positive direction. The visual angle has not been changed, and the focal adjustment not enough so to produce any very perceptible decrease in distinctness of vision. The image appears perhaps 60^{cm} or 70^{cm} distant, but this estimate is quite uncertain. The apparent diameter is of course increased. The effect of constancy in the visual angle in this case seems to be the preponderating element in determining the judgment.

The results of experiment with the apparatus just described are given in the curve A A′, of Fig. 2. The stereograph em-

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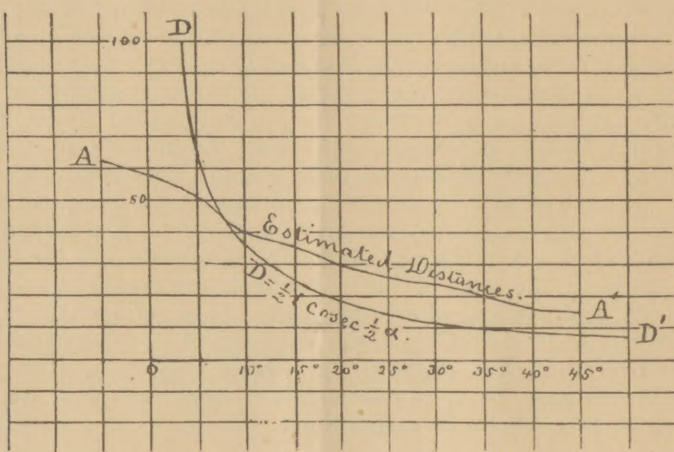

ployed was one of the full moon, so arranged as to produce reversion of relief. Distances were estimated to the edge of the concavity, which was surrounded with a uniform black surface. It is seen that the curve of theoretic distances, DD′, is cut by that of apparent distances, AA′, not far from the point corresponding to $\alpha = 7° 20'$. The stereoscope was manipulated by an assistant who varied the optic angle in irregular order, recording each value, of which I remained ignorant, and at the same time recording my corresponding estimate of distance. The curve has been constructed from the record of six independent series of estimates. In consequence of the difficulty of securing perfect dissociation between axial and focal adjustments for large positive values of the optic angle, 45° has been taken as a limit, though 80° of convergence is possible. For negative values the dissociation is not difficult, but beyond $-5°$, the image is apt to be unsteady. As might be expected, the curve is not regular, the probable error being $\pm 8^{mm}$. It is found almost to coincide with the theoretic curve for a short distance on each side of the intersec-

tion, but the judgment is much vitiated as we depart from the conditions of normal vision. Even for $\alpha = 7° 20'$ my estimate of distance was too small, and as a whole the curve shows strikingly how fallacious must be any conclusions drawn from Brewster's theory that there is a necessary connection between apparent distance and optic convergence, or, as he expressed it, that we "see distance," instead of judging it as contended by Berkeley.

The curve also shows that, under the conditions imposed, the variation in apparent distance is not very great between the limits of $-2°$ and $+5°$, within which the optic angle is included in most cases of binocular vision with lenticular stereoscopes. This explains my remark in a former article that the judgment of absolute distance is "in practice nearly, but not quite, independent of the optic angle,"[7] but is influenced rather by physical perspective. In these cases, it will be observed, the field of view is quite limited, and the optic angle not very large.

Sir David Brewster[8] noticed the strong effects obtained with convergence of visual lines by combining the images of perfectly similar patterns, recurring regularly and in great number, on large surfaces. When an extended field of view is occupied by such images, the effect of contraction in the rectus muscles seems to be more marked in comparison with that of the other elements of perspective; in estimating absolute distance there being no contrast of background and foreground to interfere. This enhancement is noticeable also when the visual lines are made to diverge, but still the positive visual angle is more important than the negative optic angle in determining the resultant effect. This is well shown by the following experiment: A large vertical surface is found upon which there are regularly recurring figures separated from each other by an interval 3^{mm} in excess of my interocular distance. Standing 50^{cm} off, in front, the images of contiguous pairs are combined by axial divergence, the optic angle being very nearly $-0° 21'$. The illusion is that of a papered wall about 3^{m} distant. Approaching them until the divergence is doubled, the wall appears about 2^{m} distant. The increase of divergence tends to produce the effect of apparent recession, but this is much more than counteracted by increasing the visual angle as the true distance is diminished, causing the externally projected image to appear to approach from the further side, instead of receding. The size of the retinal image varies in accordance with a well known law, but optic divergence slightly modifies the interpretation that would otherwise be suggested by the sensation. The apparent distance and diameter are thus made to vary together at will. Comparing the for-

mer, however, 3^m, with that expressed in the curve A A′, 58^{cm}, for the same negative optic angle, and the same real distance, 50^{cm}, it is seen that the change of conditions has produced a great change in the unconscious interpretation of the retinal image. In both cases the facts contradict Brewster's theory of triangulation. Brewster[3] himself noticed that when the combined image is small in comparison with the whole field of view, it did not appear at its calculated distance, even with convergence of axes, and to get rid of the disturbance due to comparison he resorted to large surfaces with geometrical patterns, but evidently without suspecting that optic divergence in viewing them was possible.

While the curve A A′, fig. 2, represents the result of experiment upon myself alone, similar results have been obtained from the examination of several other persons. In each case the range of uncertainty has been large, and the curve of theory has been found to be crossed by that of experiment near the point corresponding to the optic angle of normal binocular vision. Estimated distances were always greater or less than distances calculated by use of the formula, according as the optic angle was greater or less than that of normal vision at the true distance of the object regarded. For small and negative angles, not only is the departure of the curve of experiment from that of theory very rapid, but the uncertainty in the estimate of distance is increased. For larger angles the two curves approach more nearly to coincidence if the observer is well practiced and at the same time presbyopic, so that ciliary adjustment interferes less with the suggestion due to axial adjustment.

There are several considerations which interfere still further, in comparing results obtained from different observers. One person may be less accustomed than another to depend upon the sensation of muscular strain in interpreting visual perceptions. The definiteness of the interpretation becomes less as the departure from normal conditions increases. The same degree of convergence or divergence may imply greater muscular strain for one pair of eyes than for another, according to the elasticity and vigor of the muscles, or the age of the individual. Even when the real distance of the object is varied with the optic angle according to the formula so that there is no departure from the conditions of normal vision, the estimates of each observer will be found to be affected with a nearly constant error, as has been shown by the experiments of Helmholtz and Wundt.[8] This error was more than 33 per cent of the true distance in many of Wundt's experiments. In abnormal vision, furthermore, the dissociation between axial and focal adjustments is usually not instantaneous, and for strong

convergence it is very limited. The curve A A′ might perhaps have approached more nearly that of theory had I waited longer for ciliary adaptation, but this in turn involved great fatigue in the rectus muscles, especially when many experiments in succession were made, and the error was as great from this cause as from imperfect dissociation. Brewster observes, in reference to the binocular image obtained by forced convergence,[8] that "it generally advances slowly to its new position," and he speaks of "the influence of time over the evanescence as well as the creation of this class of phenomena." The nature of focal accommodation by action of the ciliary muscle was not then known—(1844).

3. A. New Mode of Stereoscopy.

The study of physiological perspective has enabled me to attain the explanation of a phenomenon first observed by Brewster, but not explained by him, and directly contrary to his theory of visual triangulation. In viewing with strong cross-vision a large plane surface on which are regularly recurring figures, such as wall paper, a phantom image of the wall is easily obtained, which appears suspended in mid-air. Of this image Brewster observes in passing, that "the surface seems slightly curved,"[9] but discusses this feature no further. His explanation[10] of the production of the phantom wall is easily understood. Let A, B, C, etc. (fig. 3), be equidistant points on the

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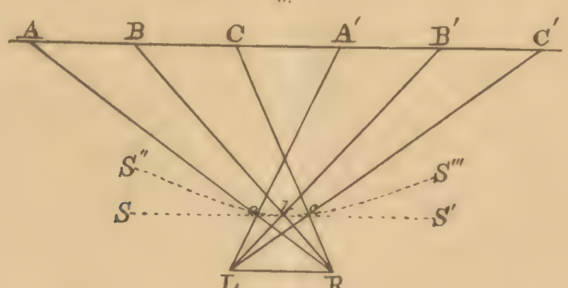

wall, in front of which stands the observer whose eyes are at R and L. If the right eye be directed to A and the left to A′, the intersection of visual lines is at *a*. In like manner, *b* and *c* are determined, and it is an obvious geometric necessity that the locus of the phantom surface must be a plane S S′ parallel to the given plane AC, assuming the line L R to be parallel to it also. It cannot be a curve S″ S‴, if the theory be true. To make the curvature apparent, the optic angle must be large; and on account of the exceeding muscular strain it in-

volves, the experiment has probably been rarely tried, and soon passed into oblivion. The curvature of the phantom surface, in a median plane passing vertically between the eyes, was rediscovered a short time ago by Professor LeConte, and soon afterward I discovered the curvature in all directions.

The effect is in no way due to intersection of visual lines, but to the opposite obliquity of vision with each eye separately, combined with the fact that the retinal surface is not plane but almost spherical at the points impressed, the center of curvature being very near the nodal point of the crystalline lens. The experiment is therefore far easier and more striking if optic parallelism or slight divergence be substituted for strong convergence, and if, instead of a wall, a pair of cards be employed, on which are perfectly similar figures, such as a pair of similar series of concentric circles. If there be difficulty in directing the eyes, an ordinary stereoscope can be used as an aid. I have devised a simple attachment for the adjustable stereoscope described in my last paper, by which one can with perfect ease thus secure stereoscopy with similar figures. Dissimilarity between the external pictures has hitherto been deemed indispensable for the attainment of true stereoscopic effects. The present method therefore, in which advantage is taken of the globular form of the eye, so far as I can learn, is entirely new. The binocular relief moreover can be reversed at will without consciously changing the relation between the visual lines, and the same pair of simi-

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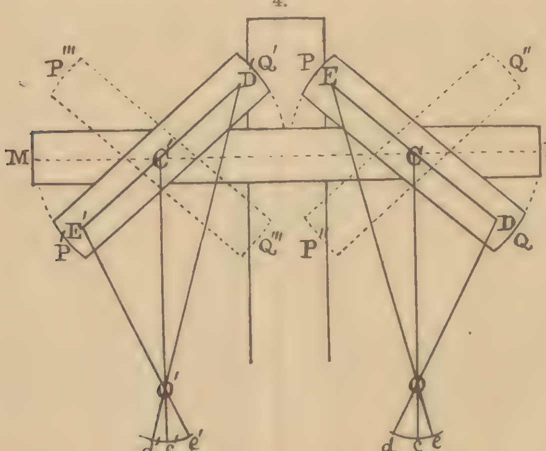

lar pictures can be examined with comfort while in form the binocular image changes from an elliptic convex shield to a flat circular plate and thence into a deep elliptic cup; the process being reversed at pleasure.

The attachment consists of an ordinary cross-bar, M N, fig. 4, which may be placed as near as convenient in front of the eyes whose optic centers are at O and O′, the visual lines being parallel and passing through points C and C′. These are the centers of the conjugate series of concentric circles, on cards whose planes are perpendicular to the principal plane of vision, and which rest on extra short bars, P Q and P′Q′. The latter are pivoted on the cross-bar so as to revolve about vertical axes passing through C and C′ respectively. Let E D and E′D′ be the horizontal diameters of the largest circles, the cards having been revolved so as to make with each other a dihedral angle opening toward the observer. Their relation to the visual lines is obviously the same as if their planes were coincident and the visual lines crossed, making the optic angle equal to the sum of N C D and M C′E′, as in Brewster's experiment. The retinal projections of E C D and E′C′D′ are *e c d* and *e′c′d′*. Since the triangles E O D and E′O′D′ are oblique, their medians divide the angles at O and O′ unequally; hence $dc > ce$ and $d'c' < c'e'$. The retinal images in the two eyes are hence dissimilar; and this dissimilarity may be made so great by increasing the angles N C D and M C′E′ that the binocular image becomes confused if the eyes are not made to play rapidly over the picture. If the attention be momentarily withdrawn from C and C′ to D and D′, the visual lines become divergent to an extent measured by the difference of the angles D O C and D′O′C′. The associated contraction of the external rectus muscles which this necessitates at once produces the sensation that habitually accompanies recession of the object binocularly viewed. The same is true if the attention be restored to C and C′, and then given to E and E′. The binocular image of the horizontal diameter must hence be perceived as a curve, convex toward the observer.

Each circle moreover must be projected upon the retina approximately as an ellipse whose minor axis is horizontal, its ratio to the major axis being readily calculable if the angle of inclination be known. But the retinal ellipses are no longer concentric (figures 5 and 6), the extent of retinal displacement depending on the extent of the minor axis in each. If the successive vertices be connected, we have two curved lines, A C B and A′C′B′. If these be binocularly combined and externally projected, since C C′ is less than A A′ and B B′, optic divergence becomes necessary in transferring the attention from C and C′ to A and A′ or B and B′. The binocular image of the vertical diameters must hence be perceived as a curve, convex toward the observer.

Let F and G (fig. 5) be points symmetrically situated with regard to the vertical diameter and hence equidistant from D

and E respectively. When the card is revolved, as in fig. 4, the distance O E exceeds O D, and hence the visual angle subtended by E G is less than that subtended by D F. Every ellipse therefore is distorted. To each eye separately the effect

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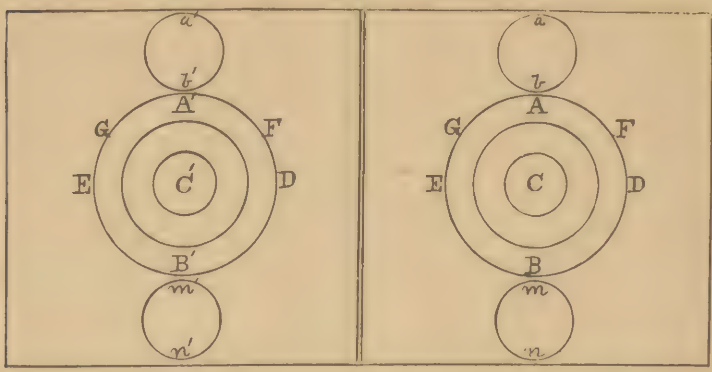

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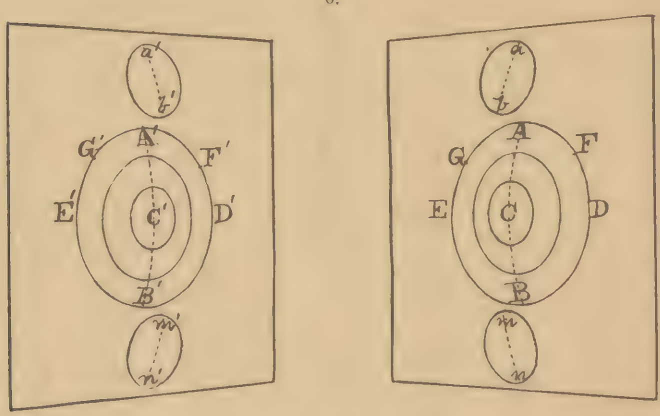

is the same as if every major axis were bent, and every point of each curve were correspondingly displaced; F and G′ are elevated, F′ and G depressed, and hence F and F′ differ in retinal latitude as well as longitude. The binocular combination, however, is perfect, although double images due to difference in retinal latitude are neither homonymous nor heteronymous. That conjugate points differing slightly in altitude can be binocularly viewed and their images combined, even when there is no horizontal stereoscopic displacement, was first shown by Professor W. B. Rogers.[11] This is one of several considerations which show that the theory of corresponding points in binocu-

lar vision cannot be accepted in any mathematical sense. In the present case the distortions of the retinal ellipses, being equal and opposite in the two eyes, are perfectly corrected in the binocular combination of each pair: the resultant curves are hence perfect ellipses.

If a pair of small circles whose vertical diameters are $a\,b$ and $a'b'$ be drawn above the large circles, the visual lines directed to their centers are similarly oblique to their vertical diameters but oppositely oblique to their horizontal diameters. The external projections of their retinal pictures are hence slightly distorted ellipses, of which the upper vertices are farther apart and the lower vertices nearer together than their centers. The binocular combination is hence an ellipse whose plane is oblique, the upper vertex being farther, and the lower vertex nearer to the observer. A pair of small circles below the large ones are binocularly combined with opposite obliquity.

No explanation is now needed to show that if the planes of the cards be revolved into the positions P″Q″ and P‴Q‴ (fig. 4), the combination of the concentric circles must present a concave surface and the obliquity of the plane of each pair of conjugate small circles, when binocularly viewed, must be reversed in sense.

Brewster's remark about the phantom wall that "it generally advances slowly to its new position"[10] is now easily understood. When E and E′ (fig. 2) are binocularly viewed, since E O exceeds E′O′, there must be dissociation between the two focal adjustments which are generally adapted to the same distance. To this must be added the necessary dissociation between axial and focal adjustments, the former being for an infinite distance, the latter for the distance O C, when C and C′ are binocularly viewed. To untrained eyes this unusual combination of muscular actions is at first a little confusing, and for a few seconds it is often difficult to decide whether the surface appears convex or concave. Despite this inconvenience, if the experiment be performed with axial parallelism, the image soon becomes clearly defined. With strong axial convergence, as in Brewster's method, the dissociation is far more difficult on account of the extreme muscular strain that is necessary.

By holding the cards, fig. 4, with their planes coincident and then drawing them apart in this plane, so that 6° or 7° of divergence of visual lines is necessitated in retaining retinal fusion, the image changes very perceptibly from that of a flat plate to that of a shallow concavity. By cross vision the opposite is obtained; but not so strikingly, for the muscular strain of 7° of divergence I find to be as great as that of 60° or 70° of convergence.

If vertical lines, $a\,n$, $a'n'$, fig. 5, be combined binocularly

they appear as a tangent to the curved surface, and this pierces the planes of the small ellipses, passing through each at its center. Any inaccuracy in either drawing interferes with these results and is at once manifested in the binocular picture.

40 W. 40th st., New York, Feb. 25, 1882.

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[1] This Journal, Nov. and Dec., 1881.
[2] Edinburgh Transactions, vol. xv, Part III, p. 360.
[3] Brewster on the Stereoscope, London, 1856, pp. 50–100.
[4] This Journal, II, vols. xx and xxi.
[5] Am. Journal of Photography, vol. v, p. 114.
[6] Helmholtz, Optique Physiologique, p. 827.
[7] This Journal, Dec., 1881, p. 447.
[8] Optique Physiologique, p. 824.
[9] Brewster on the Stereoscope, p. 91.
[10] The same, p. 95.
[11] This Journal, II, vol. xxi, p. 181.
[12] Elementary Physiology, Macmillan & Co., p. 280.

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Art. XXXVII.—*Notes on Physiological Optics*; No. IV; by W. LeConte Stevens.

1. On Voluntary Control of Focal Accommodation.

In my last paper the visual effect of associated muscular action of the two eyes was discussed, the investigation having led to the discovery of a new mode of stereoscopy. Experiments have also been made on the effect of muscular action in a single eye.

If the axial adjustment and visual angle be kept as nearly as possible unchanged while the gaze is directed upon a luminous surface, such as the globe of a gas lamp, it is very easy to throw the crystalline lens of each eye out of focus, accommodating for a nearer point, so that the object appears blurred in consequence of the production of diffusion circles on the retinas. I find it possible in this way to exercise extreme contraction of the ciliary muscle at will. It is impossible to avoid slight associated contraction of the internal rectus muscles, but this may be controlled to such an extent that the only effect noticeable is unsteadiness of one eye, while the other is directed to some previously clearly defined object. The effect is first an encroachment of diffusion circles upon the surrounding retinal area as well as upon the image itself, producing dimness of vision and hence apparent enlargement of the object which is necessarily ill-defined. But as the ciliary contraction increases, the object apparently diminishes in size to a marked extent, sometimes almost to half its previous area, while the halo due to diffusion circles does not widen, but on the contrary grows narrower. This is due to the contraction of the pupil, the effect being that of using a new stop in front of the lens, as it thickens and acquires more "depth of focus," while its theoretic focal length is less. The pupillary contraction can be noted by an assistant and approximate measures be made of the change in diameter of the opening. This pupillary change very quickly follows the contraction of the ciliary muscle. The area of my pupil has thus been repeatedly diminished, within two seconds of time, from 12 sq. mm. to 1·2 sq. mm. The estimated distance of the object is very uncertain. It may be judged nearer because of the sense of muscular contraction, or more remote, because of the dimness due to imperfect focalization. The only perfectly clear perception therefore, is that of marked dedecrease in area, far greater than can be referred to mere encoachment of diffusion circles. The distance between retina and lens remains sensibly constant, hence the perception is an illusion due mainly to abnormal muscular conditions.

Since in normal vision the contractions of the ciliary and internal rectus muscles are associated, the considerations just expressed show why so acute an observer as Wheatstone should have noticed the apparent decrease in size of the binocular image when strong convergence of visual lines was induced by pulling forward the arms of his reflecting stereoscope,* but failed to note the corresponding variation in apparent distance, though he observes that the image seems changed in position, but does not say whether the change is that of increase or decrease in distance.

2. Effect of Muscular Effort on Retinal Sensitiveness.

The estimation of size and distance, when there is strong, muscular effort, depends in some measure upon the part of the retina on which the image is formed, variations being more noticeable when the central part is impressed. In a former paper† I briefly noted the following experiment with Wheatstone's stereoscope. Let A and A′ (fig. 1), be a pair of conjugate pictures adjusted so that the reflected rays, *m i* and *m′ i′*, are sensibly parallel. If the visual lines of the eyes receiving these rays coincide with them, the image appears in front, combined in full relief, with apparent diameter and distance slightly magnified, the contraction of the internal rectus muscles being less than normal. Let the visual lines now be forcibly crossed. The retinal images, *i* and *i′*, are no longer upon corresponding

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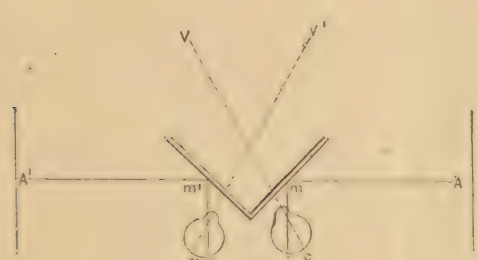


2.

retinal points: the external projections of them hence appear separate and without relief. For the subjective binocular eye (fig. 2), the two visual lines are combined into the median line *c*M. By indirect vision, the image of A appears on the right of the median, and that of A′ on the left, each diminished in size on account of the disturbance of ordinary retinal sensation by strong muscular contraction. This apparent diminution is greater than

* Philosophical Magazine, 1852, p. 508. † This Journal, Dec., 1881, p. 450.

can be explained by encroachment of diffusion circles at the edge of the retinal image, or on the supposition that the antero-posterior diameter of the eyeball has been slightly diminished by pressure on opposite sides of its elastic sclerotic coat. For reasons already given, the apparent distance is not very determinate. Without changing the convergence, let the eyes be rolled to the left until A′ is seen with the left eye in the direction of its visual line. The image of A, seen still more obliquely, now appears slightly larger and dimmer, while A′ is still smaller. The apparent variation in size is thus independent of binocular combination.

3.

In such experiments one great advantage attained by using Wheatstone's stereoscope is, that by keeping the plane of each picture perpendicular to the arm that carries it, any distortion of perspective that might be due to changing the direction of the visual lines is reduced to a minimum. Making allowance for such disturbance, the visual effects just described may be obtained by means of fig. 3. A pair of circles, 2^{cm} or 3^{cm} in diameter are drawn 8^{cm} or 10^{cm} apart, their centers, A and A′, being similarly marked. Below A′ is another circle A″, equal in size. By cross-vision the images of A and A′ are combined, forming a diminished ellipse, each circle being seen obliquely; the minor axis is parallel to the interocular line. The monocular images on the two sides are sensibly circular, and larger than the central image, but smaller than either given circle as seen by normal vision. Even though distinct vision be attained under these conditions, the ciliary muscles are in a state of strong involuntary contraction. On closing one eye, these muscles become relaxed, while the direction of the visual line for the eye remaining open is easily kept unchanged. The elliptic image then apparently recedes, and in doing so it grows larger; but an interval of one or two seconds may elapse before normal monocular vision is restored. This process can be

reversed and made as gradual as is desired by slowly increasing the convergence of visual lines and carrying it beyond the point necessary for mere binocular combination. If the pair of circles just mentioned be employed, as soon as double vision is produced, the two interior images are perceived to be smaller than the others. They continue to grow smaller and apparently nearer to the observer until they coincide in the middle, and the diminution continues after they have been made to pass each other. In fig. 3, these phenomena may be noticed if the reader will combine the images by cross vision, placing the face as near as possible to the page. The angle of convergence should be at least 60°. The elliptic monocular image of A'' is perceived as suspended in mid-air. Its apparent position is not quite so nearly determinate as that of the binocular image above it, but it appears about as small and near. The right eye image, seen by indirect vision, has a much greater apparent area than the left eye image seen by direct vision.

In connection with the phenomena of optic divergence, this series of experiments proves that not only is convergence of visual lines not necessary for binocular vision and for the localization of objects in the field of view, but binocular vision itself is not necessary for the production of change in such localization. The mental impression received from the binocular image is clearer and the resulting judgment is much more definite, but no geometric method of determining the place of the external image is capable of covering the facts.

The experiments described indicate a closer connection between the nerves of the ciliary muscle and those of the retina than has been commonly supposed. Through the sympathetic system of nerves the mere action of light on the retina excites reflex muscular contraction of the iris. Without the agency of light, this may be effected by ciliary contraction alone, which is always accompanied with diminution in size of the pupil. It is by no means impossible that, through the sympathetic system, ciliary contraction should modify also the impression conveyed to the brain from the retina, while the size of the actual retinal image is unchanged. The interpretation is dependent upon the excitement of several different nerves; the thing interpreted is the product of forces from without that operate in accordance with well known mathematical laws. Helmholtz has determined approximately the velocity of propagation of a nerve impression, but beyond this there has been little success in bringing the physiology of sensation within the domain of mathematical law.

3. Relation of Axial Adjustment to Focal Accommodation.

The focal and axial adjustments of the eyes being necessarily dissociated to some extent, if clear vision is had with the stereoscope, but this dissociation being abnormal, it follows that the range of focal accommodation possible for a given pair of eyes should not be the same for different values of the optic angle. To ascertain the degree of disturbance thus induced, a series of measurements has been made with the assistance of Dr. J. H. Shorter, of the New York Eye and Ear Infirmary. In the preliminary examination it was found that at the distance of 9^m it was possible to read Snellen's test-type intended for average vision at 6^m. By the usual formula, therefore, my acuity of vision is $\frac{9}{6}$. With visual lines approximately parallel, there is hyperopia to an extent not exceeding one dioptric.* This is capable of correction, therefore, with a bi-convex lens whose focal length is 1^m. But when the visual lines are diverged 5°, latent hyperopia to the extent of nearly one dioptric more is revealed by the use of appropriate test-lenses, while the near-point of distinct vision is found, by means of test-type, to have receded proportionally. This fact could not well have been ascertained otherwise, except by the use of atropine, with its well known disagreeable consequences. The experiment shows that, in the present case at least, full relaxation of the ciliary muscles is not attained by making the visual lines parallel. For emmetropic eyes it is usually assumed that the passive condition is that of parallelism. The theory of negative accommodation advanced by Von Graefe† and Weber has not been generally accepted.

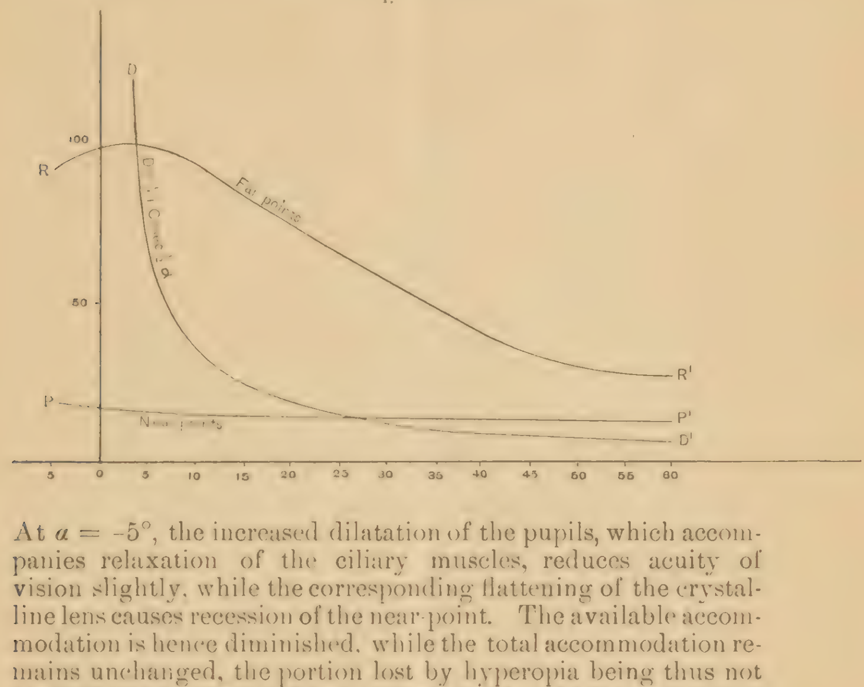
In experimenting upon range of accommodation I employed a pair of narrow cards on which were vertically printed similar specimens of test-type, intended to be read with average acuity of vision at the distance of 67^{cm}. On a large sheet of paper, covering a table, a median line was drawn, terminating at the edge in front of the mid-point between the eyes. Making due allowance for the distance of these from the edge, pairs of lines were drawn, along which the visual lines could be directed at will. Upon them the cards were placed so that binocular fusion of images could be secured for fixed values of the optic angle, to which the directions of the lines had been adapted. Along these the cards were made to recede and approach until the farthest and nearest points were found at which distinct binocular vision was possible, myself being kept ignorant of the record taken until the whole series of measurements had been completed.

* For dioptric system of measuring the refraction of the eye, see Carter on Eyesight, pp. 43–47; Macmillan, 1880.

† Soelburg Wells, Diseases of the Eye; Appleton, 1881.

The result is shown in the upper and lower curves of fig. 4, in which distances are expressed in centimeters, for successive values of the optic angle, taken as abscissas: RR′ is the curve of far-points, PP′ that of near-points, and DD′ that of distances of the optic vertex determined by the intersection of visual lines. The latter curve crosses PP′ near the point corresponding to $\alpha = 25°$, showing that, for values of the optic angle greater than this, distinct vision of an object as near as the optic vertex is impossible. This limiting value of the optic angle becomes still smaller with increasing age. The range of available accommodation in the present case is seen to be greatest when $\alpha = 3° 30'$. This is about the angle enclosed by the visual lines when a single point is seen at the distance of 1^m.

4.



At $\alpha = -5°$, the increased dilatation of the pupils, which accompanies relaxation of the ciliary muscles, reduces acuity of vision slightly, while the corresponding flattening of the crystalline lens causes recession of the near-point. The available accommodation is hence diminished, while the total accommodation remains unchanged, the portion lost by hyperopia being thus not available when convex glasses are not used. As the optic angle increases beyond 3° 30′, the distance of the far-point rapidly diminishes, so that, at $\alpha = 60°$, more than three dioptrics of temporary myopia have been induced. It was found that at $\alpha = 4°$, the weakest convex lens impaired vision; this amount of convergence is hence sufficient to completely mask my hyperopia, stronger convergence much more than masks it, parallelism makes it manifest, while divergence reveals that which is latent

and not otherwise discoverable except by temporarily paralyzing the ciliary muscle.

4. Relation of Pupillary Area to Axial Adjustment.

Pupillary change usually accompanies change of accommodation, though not an exact index of it. During the recent experiments measurements were made of the diameter of the pupil, the intensity of the light being kept as nearly constant as possible. When axial convergence takes place, the objects being as far as can be seen distinctly, the pupils first contract, then relax perceptibly, but do not recover their previous size even after binocular vision seems perfectly distinct. When axial divergence takes place, the objects being near, the pupils dilate noticeably, and then diminish very slightly. From the following table of measurements it is seen, on comparison with the curves of fig. 4, that for the near-point, when $\alpha = -5°$, the pupil is larger than for the far-point when $\alpha = +3° 30'$. If α exceed 45° there is no perceptible variation of the pupil when far-point and near-point are successively regarded.

Pupillary areas, in square millimeters.

Optic angle, --------	$-5°$	$+3° 30'$	$+25$	$+60°$
Maximum area, ----	12·4	7·0	3·0	1·2
Minimum area, ------	8·2	4·2	1·7	1·2

For a given value of the optic angle it is quite probable that pupillary variation is proportional to change of ciliary contraction; but the table shows that it is much limited by association with variation of contraction in the rectus muscles. The dissociation between focal and axial adjustments is but rarely complete when the convergence is strong. If diffusion circles on the retina are very small, while the image is bright, we easily fail to notice them, and it is not easy to determine when perfect distinctness of vision passes into slight indistinctness. The range of perfect accommodation is doubtless much narrower even than is indicated in the curves of fig. 4, granting that the observer's eye is perfectly free from such defects as would produce permanent error of refraction. Since the size of diffusion circles depends upon the area of the pupillary opening as well as upon error of refraction, permanent or temporary, and since distinctness of focalization is indispensable to success in binocular vision, either with or without the stereoscope, the bearing of these experiments upon the theory of binocular perspective is obvious. Though many of them are straining, they are performed to some extent, more or less unconsciously, by every one who uses a stereoscope that is ill-adapted to his eyes and stereographs that are improperly mounted.

5. The Operation of the Will in Vision.

In reference to the operation of the will upon the muscles of the eyes, Helmholtz states that we are limited to efforts for the attainment of single and distinct vision. The French edition of his great work on Physiological Optics received the last corrections of the author, and from this the following quotations must be made:

"Il faut remarquer en général que, dans toutes les mouvements volontaires, notre volonté ne tend jamais qu'à atteindre un résultat extérieur nettement déterminé et perceptible par lui-même."—*Opt. Phys.*, p. 613.

"Il résulte de ces faits que la relation qui existe entre les mouvements des deux yeux n'est pas commandée par un mécanisme anatomique, mais qu'elle se modifie, au contraire, sous l'influence de notre volonté; la seule limite réside dans le fonctionnement de notre volonté que nous ne savons pas appliquer à un but autre que celui de voir les objets simples et nettement."—*Opt. Phys.*, p. 617.

The experiments just described show that the will may be directed to the attainment of other ends than single and distinct vision. Although not affected with any paralysis or other weakness of the muscles of the eyes, and hence never troubled with involuntary double vision, I find it quite possible at any time to converge or diverge the visual lines without using any external points of fixation, thus securing two separate monocular images of a given object, keeping these apparently separated at any desired angle within certain limits, and fixing the attention alternately upon either image separately, at pleasure. A door-knob, for example, may be distinctly seen double by diverging the visual lines a few degrees, and either separate image may be thus examined. Discarding the use of one eye, this externally projected image may be varied at will, in sharpness of definition, apparent size, and apparent distance, by strong voluntary ciliary contraction. This power was of course never suspected until revealed by special experiment; but there is no reason to suppose that it is not possessed by most others whose eyes are healthy.

6. Binocular Vision and Binaural Audition.

The discoveries in reference to binaural audition, made during the last few years independently by Professor Silvanus Thompson,* of England, and Professor A. M. Mayer, of this country, are interesting, not only as additions to our knowledge of physiological acoustics, but also in relation to the phenomena of

* Philosophical Magazine, Oct., 1877, Nov., 1878, and Nov. 1881.

physiological perspective. The localization of sounds has been found to be much affected by the mode in which the waves are conveyed to the separate ears. The same tone may be perceived as if produced at the back of the head, or from the two sides, or from a point obliquely in front, while the position of the true external source is unchanged, the perception being involuntary while the conditions are adjusted at will. The judgment of distance by the ear is far more uncertain than by the eye, there being no other criterion than the degree of energy of the vibrations which give rise to sensation; but the perception of direction may be modified by imposing special conditions, such as fatiguing one ear with a given tone and then listening to the same with both ears. For a fixed position of the eye, the perception of direction may be modified at will by methods already described, or by pressing upon the eyeball, while that of distance is also subject to variable conditions. Although the binaural estimate of direction and distance may be made less uncertain by properly adjusting the position of the head to the wave front, or, as in the case of the lower animals, by directing the two ears at will toward the source of sound, no one has attempted to apply geometry to the binaural localization of sounds. Its application to binocular vision is now found to be wholly unreliable in the very department for which it was deemed most satisfactory, that of stereoscopic perspective.

7. Effect of Experience in Vision.

Assuming that focalization is distinct, the position of the external focus, conjugate to that on the retina, is calculable, but for distances exceeding a few metres the variation in thickness of the crystalline lens becomes so slight that as an element in forming judgments of distance it may be practically discarded. We are justified in saying therefore that there is generally no recognizable relation between the character of a retinal image and the distance of the object pictured, any more than between the composition of a sound image and the position of its external source, except so far as personal experience serves to establish it. Experience, both visual and auditory, begins in infancy, long before the power to analyze sensations is developed, and in putting an interpretation upon these, association is the chief determinant. Strong contraction in the ciliary and internal rectus muscles is unconsciously associated with nearness of the point of fixation, and this is pictured upon the yellow spot of the retina. No one whose eyes are healthy has any consciousness of possessing any retina except in relation to external objects, or any tympanum except in relation to

aerial disturbances, or any ciliary or rectus muscles except in relation to variation in position of the point on which attention is fixed through the medium of retinal sensation. We might expect therefore that unusual muscular conditions would cause misinterpretation regarding the whole field of view, but especially those parts which correspond to that portion of the retina which is most frequently impressed and most sensitive. Under strong muscular contraction, arousing abnormal vision, it is hence not remarkable that an object pictured on the central part of the retina should be misjudged as smaller and nearer than one of the same real size and distance pictured upon marginal portions; and that the illusion should disappear as soon as the muscles are relaxed and normal conditions thus restored.

There is one psychologic aspect of the present investigation of which I have not been unmindful, particularly in reference to the effects of extreme voluntary ciliary contraction. In recording the results of even normal vision every astronomer applies what experience has taught him to be his personal equation. In the present experiments in abnormal vision, some of which are not easy, the personal equation cannot well be ascertained. While there has been given what I believe to be a correct account of phenomena actually observed and not consciously anticipated by myself, there must remain, for a time, an element of uncertainty, how much may be due to personal peculiarity and how much to qualities belonging generally to the human eye. The most careful of observers may be occasionally mistaken in attributing to physical conditions what is really the subjective effect of unconscious expectant attention. The very consistency of these phenomena with the empirical theory of vision might cause a predisposition to certain perceptions that might not be so successfully attained by one who is thoroughly committed to the opposite school of thought.

8. Theories of Binocular Perspective and Relief.

The reasoning already expressed suggests additionally the futility of attempting to refer our binocular perceptions exclusively to any single condition such as the external intersection of visual lines or the internal recognition of double images in one part of the field of view while another part is scanned, applicable as these conditions may be when considered simply as elements that may at times enter into our interpretations. In their application to stereoscopy the physiological elements of perspective imply no contradiction to the laws of mathematical perspective which must be applied in constructing the stereo-

graph. The relative distances in the drawing being fixed, variation in muscular sensation may modify the imagined scale of measurement, while ratios are sensibly the same as before. The beautiful results attained by Professor W. B. Rogers* in determining the form of the binocular resultant are applicable to projecting lines that intersect, and not necessarily to visual lines, since the perception is attained when these are divergent. The same remark is true of Helmholtz' admirable mathematical discussion† of the stereoscope, in which he makes no provision for optic divergence, although elsewhere he refers to the possibility of stereoscopic vision by this method. It is but due him to state that, under the conditions assumed, he closes his discussion with the remark, "These conditions are not always fulfilled for the photographic proofs and the stereoscopes of commerce."

Associated muscular action, to which special prominence has been assigned in the present series of papers, is in like manner incapable of explaining all the phenomena of stereoscopy. Much stress has been laid upon it because it covers all the facts that have hitherto been referred to visual triangulation, and additionally those of optic parallelism and divergence, assuming that the stereoscopic displacement on the card is large enough to permit the perception of double images in the background when the foreground is viewed, and that the time of illumination is sufficient to allow free play of the eyes. The perception of stereoscopic relief is possible when the stereoscopic displacement is so minute that no sensible motion of the eyes is possible and no duplication of any points in either foreground or background can be perceived with even the keenest vision. The stereograph has been repeatedly examined, and relief distinctly perceived, successively with natural and reversed perspective, when illuminated by the electric spark, the duration of the retinal impression being thus reduced to a minute fraction of a second. Undoubtedly the play of the eyes often enables us to become at once sure of interpretations that would be enveloped in uncertainty until after several trials by momentary illumination. In ordinary vision we rarely ever notice the duplication of images on the hither and farther side of the point of fixation, partly because the attention is apt to be confined to the point fixed, but also because other points are not distinctly focalized upon the retinas, and the majority of these imperfect images fall on retinal parts that are not central and hence are deficient in sensitiveness.

It has been suggested‡ that the perception of relief is *by*

* This Journal, II, vol. xxi, pp. 91, 173, et seq.
† Optique Physiologique, p. 842, et seq.
‡ LeConte, Sight, p. 151; Appleton, 1881.

means of double images, and that the mind *instinctively* distinguishes between those made by objects that are respectively farther and nearer than the point fixed. This last proposition would be hard to demonstrate experimentally, but even when the attention is not specially given to the double images, these may, and probably do, play an important part as elements in the unconscious formation of judgments. Again it has been stated* that we see *at long and short distances at the same time*, because the retina has thickness and transparency, and images are focalized at different depths beneath its surface. Here again we can neither affirm nor deny in answer, although recognizing the fact that the crystalline lens, being one of short focal length, has in consequence of spherical aberration considerable "depth of focus." It is moreover fluorescent, irregular in structure, and imperfectly centred. Perfectly sharp focalization is hence impossible, as shown in the radiated appearance of stars and the irradiation about any brilliant surface like that of the crescent moon. These optical defects may be in some respects advantageous in ordinary vision. If Towler's theory be true, though not demonstrable, it may partly explain the possibility of binocular combination when the differences between the two pictures are so minute that the perception of double images in any part of the binocular field is impossible. Some idea can be formed of the minuteness of the stereoscopic displacement actually necessary when we consider that Mr. Warren De la Rue succeeded in obtaining a stereograph of the sun, from which by stereoscopic vision, the ridges of the faculæ could be perceived in sharp relief. On the stereograph of the moon, to which reference has been more than once made, the elevation of mountain ranges and solitary peaks, and even the inequalities of the supposed dead sea bottoms can be clearly seen. The crater Copernicus and the lunar Apennines stand forth particularly boldly, and the ridge that divides the bed of the heart-shaped "Sea of Serenity" can be easily traced. Any one who has undertaken the preparation of a stereograph with the pencil or pen knows how very difficult it is to avoid the production of roughness in the combined image at places where smoothness is desired. No two impressions from the same type can be taken that will not present some inequalities when stereoscopically examined, and no two groups of type representing the same sentence can be so accurately adjusted as not to betray imperfection when subjected to this searching test.

In what has been said of "corresponding retinal points," no mathematical meaning has been assigned to this expression, and obviously none can be. It is generally thought, but has not been experimentally demonstrated, that each rod or cone on

* Towler, The Silver Sunbeam, p. 310; E. & H. T. Anthony, 1879.

one retina has its mate in the other, and when such a pair are simultaneously impressed they convey but a single impression to the brain. If in each eye the retinal image is so small as to cover but a single rod or cone, we have thus the so-called *minimum visibile*, and by calculating the retinal area corresponding to the smallest object that can be separately seen at a given distance the diameter of such a retinal element has been estimated. There are strong reasons however for doubting the validity of this theory of corresponding retinal points. All we can affirm is that experience, acquired individually and probably with exceeding rapidity in consequence of inherited tendencies, has taught us to interpret retinal sensations that are slightly different in the two eyes, as the signs of an external object possessing three dimensions in space, when the images are made upon parts of the concave surfaces that bear to each other the mathematical relations imposed by the presence of such an object in normal binocular vision. Sensation is never confined to a point, or pair of points, but only to areas. If we grant the correspondence between small retinal areas, simply as the result of experience rather than as a truth in anatomy, it is not hard to understand how a pair of slightly dissimilar images may produce upon the brain a modified impression, even when no duplication in any part can be perceived. It may be quite possible for the *quality* of a retinal sensation to be modified while the additional impression, apart from that which it modifies, would be imperceptible. From what we positively know regarding sensation in the ear, we are justified in applying similar reasoning tentatively to sensation in the eye.

It is well known that no perfectly simple sound is produced by the voice, or indeed by any musical instrument; but that the difference in quality between two tones nominally the same from different sources is due to minute modifications upon sensations, corresponding to attendant waves, most of which are too faint to be perceived without instrumental aid. A well trained ear however is able to pick out some of these overtones, even when two different voices are singing nominally in unison. The brain of the listener receives two sound-images, each complex, that approximately coalesce. By special attention he singles out a few musical salient points and thus perceives double images in the background; but the great majority of the attendant modifying tones are so faint and so inextricably mixed together that it is impossible to do more than perceive their resultant effect upon the fundamental about which they are grouped. The rich combination of all stands out in strong musical relief, when compared with the impression from either voice alone, or with the sweet but thin sound of the tuning fork that sings forth the same fundamental pitch.

This comparison relates to the combination of sensations, whatever may be the cause of dissimilarity among the components of the group. In the case of audition the modifications imposed upon the sensation are due to combination among waves varying in length from a small fraction of an inch to many feet. The graphic representation by curves is an arbitrary but exceedingly convenient method of indicating the quality of a sensation from what we know of the physical character of its proximate cause, however complex this may be. Waves of light are far more minute than those of sound, and we have no evidence of sensible interference among them in the production of binocular vision. We have not the data from which a binocular image can be graphically expressed as a curve, and thus compared with one of the monocular components. But the facts that are forced upon our attention suggest kinship between the *modes of sensation* in the two cases. The group of light images in the one eye forms a sum total that is different from that in the other, and the sensations they arouse are simultaneously conveyed to the brain. Experience comes to our aid, and the modified resultant is instantly recognized, even though we may be unable to perceive separately the minute modifications which give character to the two main components that form it.

I doubt therefore the anatomical theory of corresponding retinal points, and regard that of partially correspondent retinal areas as a substitute more in accordance with observed facts. This correspondence moreover must be considered merely the effect of association resulting from oft repeated experience, an association that is very quickly established, although a newborn infant does not seem to possess the power of single vision at birth. Such retinal areas should be considered as not sharply defined, as is shown by the phenomena of irradiation. A point impressed becomes the center of a minute area of disturbance, just as on the skin a point pricked with a pin is a center from which pain is quickly extended to an area. A group of such retinal areas are of necessity partially coalescent. The absence of sharp definition on the retina is one condition on which stereoscopy becomes possible through the apparent blending of images that are really dissimilar. If the dissimilarity is but slight, clear relief is perceived without the production of any sensible duplication of images in any part of the binocular picture.

In the light of these facts it is seen that the explanation of stereoscopy with perfectly similar figures, as given in my last paper,* was incomplete. It was known to be so at the time it was given, but my intention was to present only a geo-

* This Journal, April, 1882, p. 297.

metric discussion and reserve the remainder for a new paper. In performing the experiment for the first time, confusion is generally experienced; after a few moments the form of the binocular resultant is clearly perceived, and in subsequent trials the perception is attained much more quickly, all parts of the image being sensibly equally distinct at the same moment, if the inclination of the cards be not great. It is only when this inclination is considerable that play of the eyes becomes necessary, and the associated exercise of the rectus muscles thus furnishes suggestions that are complementary to the modification of retinal impressions just discussed. There are further experiments in regard to this mode of stereoscopy by momentary illumination, which I hope to make and publish at some future time.

New York, 40 W. 40th St., April 3d, 1882.